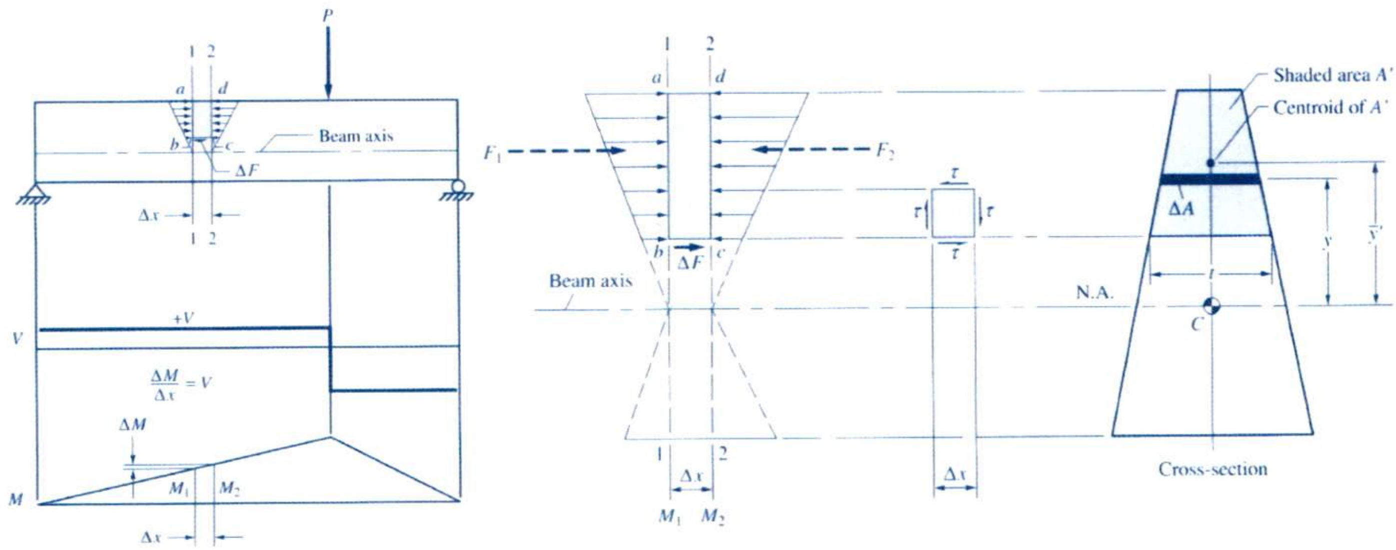
14-5

Shear Stress Formula For Beams

- In addition to withstanding applied bending moments, beams also need to have the ability to resist shear forces.
- Shear forces along the beam are also generated by applied loads and actually try to "rip the beam apart." If a beam cannot resist the shear forces a failure occurs.
- Typically, shear is not the limiting factor in design, but it must still be checked. For materials weaker in shear (concrete, wood) it may be the control parameter for the design.

Derivation of Shear Stress Formula



$$F_1 = \Sigma \sigma \Delta A = \Sigma \frac{M_1 y}{I} \Delta A = \frac{M_1}{I} \Sigma y \Delta A$$

$$F_2 = \Sigma \sigma \Delta A = \Sigma \frac{M_2 y}{I} \Delta A = \frac{M_2}{I} \Sigma y \Delta A$$

By horizontal Equilibrium $[\Sigma F_x = 0]$

$$F_1 + \Delta F - F_2 = 0 \Rightarrow \Delta F = F_2 - F_1$$

$$\Delta F = F_2 - F_1 = \frac{M_2 - M_1}{I} \Sigma y \Delta A = \frac{\Delta M}{I} \Sigma y \Delta A$$

Shear Stress

$$\tau = \frac{V}{A_s} = \frac{\Delta F}{t \Delta x} = \frac{\Delta M}{\Delta x} \frac{\Sigma y \Delta A}{I t}$$

$$\frac{\Delta M}{\Delta x} = V$$

and denote $\Sigma y \Delta A$ by Q

$$\boxed{\tau = \frac{VQ}{It}} \qquad (14\text{-}10)$$

The shear stress at any location on a cross-section of a beam (and at any point along the beam's length) can be found through the general shear stress equation.

$$\tau = \frac{VQ}{It}$$

Where,

τ = the shear stress at a point in a given section of the beam

V = the shear force at the given section

Q = the first moment of the area A′ about the neutral axis, $Q = A' \bar{y}'$

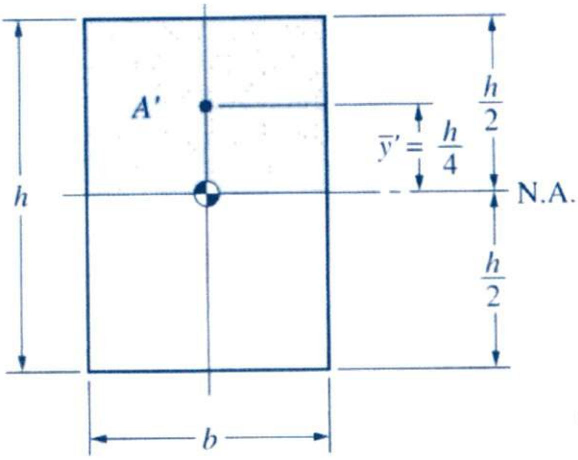
A′ = the part of the areea in the cross-section above (or below) the horizontal line where the shear stress is to be calculated.

$\bar{y}'$ = the distance from the neutral axis to the centroid of the area A′

I = the moment of inertia of the entire section with respect to the neutral axis (the same I as in the flexure formula)

t = the width of the cross-section at the horizontal line where the shear stress is being calculated

Maximum Shear Stress in a Rectangular Section



The maximum value of Q occurs at the neutral axis

$$Q = A'\bar{y}' = \left(b \times \frac{h}{2}\right)\left(\frac{h}{4}\right) = \frac{bh^2}{8}$$

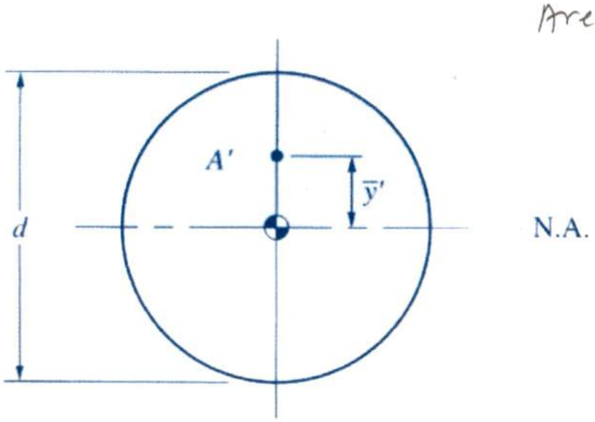
The maximum shear stress,

$$\tau_{MAX} = \frac{VQ}{It} = \frac{V\left(\frac{bh^2}{8}\right)}{\left(\frac{bh^3}{12}\right)(b)} = \frac{3V}{2bh}$$

and

$$\boxed{\tau_{MAX} = 1.5\,\frac{V}{A}} \qquad (14\text{-}11)$$

Maximum Shear Stress in a Circular Section



Area of a semicircle (Table 7-2)

$$A' = \frac{\pi d^2}{8} \qquad \bar{y}' = \frac{2d}{3\pi}$$

$$Q = A'\bar{y}' = \left(\frac{\pi d^2}{8}\right)\left(\frac{2d}{3\pi}\right) = \frac{d^3}{12}$$

$$\tau_{MAX} = \frac{VQ}{It} = \frac{V\left(\frac{d^3}{12}\right)}{\left(\frac{\pi d^4}{64}\right)(d)} = \frac{16V}{12\left(\frac{\pi d^2}{4}\right)} = \frac{4V}{3A}$$

$$\boxed{\tau_{MAX} = \frac{4V}{3A}} \qquad (14\text{-}12)$$

Example

Calculate the shear stress for the beam shown

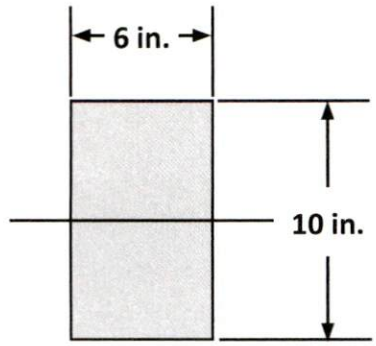
(a) at the neutral axis

(b) 3 in. away from the neutral axis

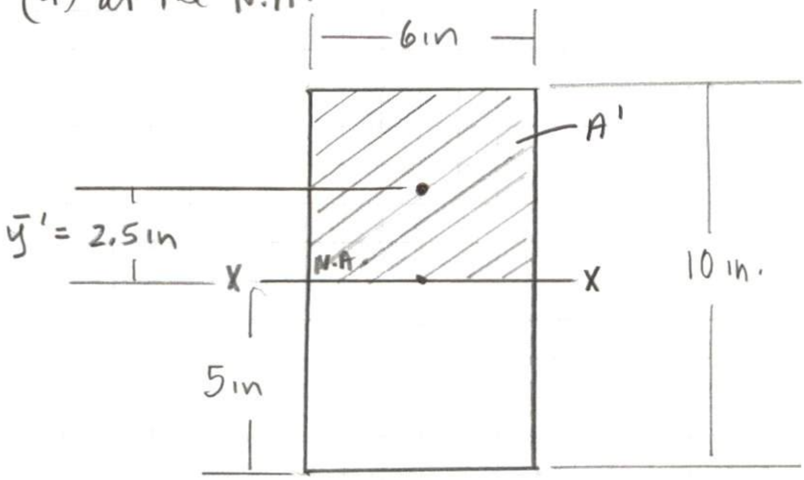
(c) at the outside surface

The shear force on the beam is, V = 50 kips

Solution.



(a) at the N.A.



$V = 50$ kip

$$I = \frac{bh^3}{12} = \frac{(6\text{ in})(10\text{ in.})^3}{12} = 500\text{ in.}^4$$

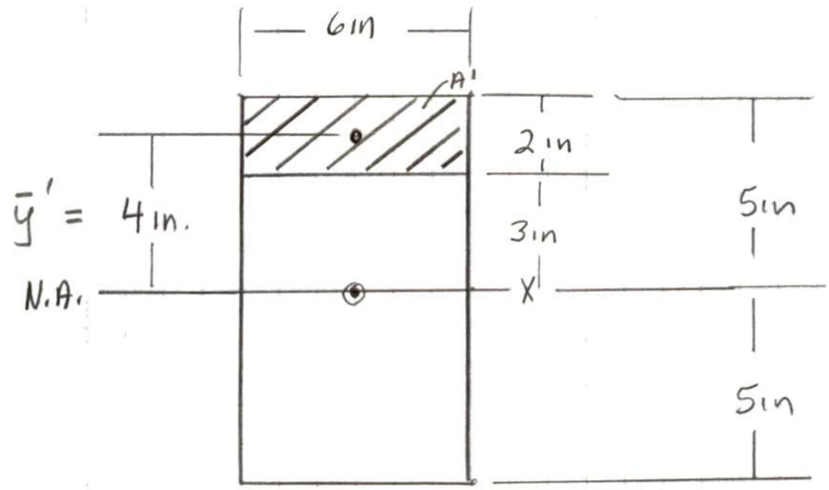
$t = 6$ in.

$$Q = A'\bar{y}' = (6\text{ in})(5\text{ in})(2.5\text{ in.}) = 75\text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(50{,}000\text{ lb})(75\text{ in.}^3)}{500\text{ in.}^4\,(6\text{ in})}$$

$$= \underline{\underline{1250}}\text{ psi}$$

(b) Shear stress at 3 in away from the N.A.



$$Q = A'\bar{y}' = (6\text{in})(2\text{in})(4\text{ in.}) = 48\text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{50{,}000\text{ lb }(48\text{ in}^3)}{500\text{ in.}^4\ (6\text{ in})}$$

$$= 800\text{ psi}$$

(c) Shear stress at the outside surface

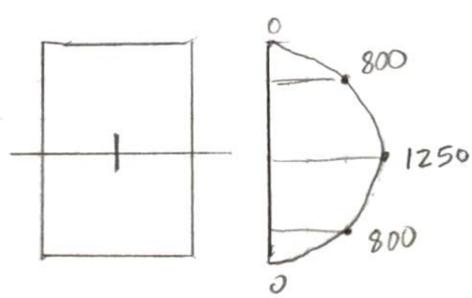
Above the outside surface there is no area

$$Q = 0$$

$$\tau = \frac{VQ}{It} = \frac{50{,}000\text{ lb }(0)}{500\text{ in.}^4\ (6\text{ in})} = 0\text{ psi}$$

Note

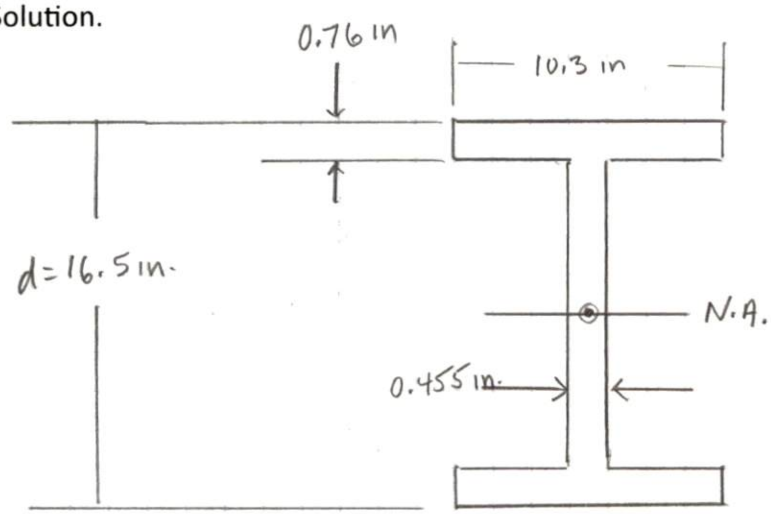
The shear stresses along a beam's cross-section are greatest at the neutral axis and vary to a minimum of zero at the outside surface.



Example

Calculate the shear stresses at the neutral axis and at the midpoint of the flange in a W 16 x 77, if the shear force is 100 kip.

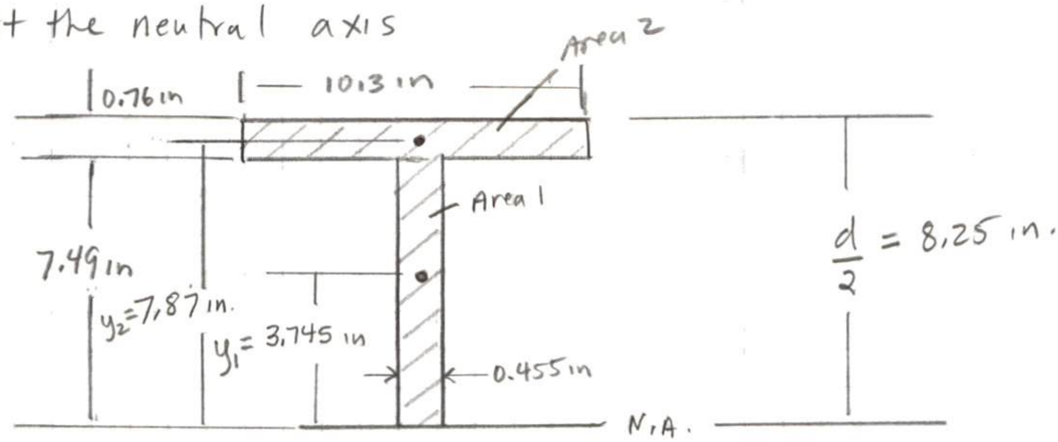
Solution.



$V = 100$ kip

$I = 1110$ in.^{4}

(a) At the neutral axis



$$Q = Q_1 + Q_2 = A_1 y_1 + A_2 y_2$$

$$= (0.455\text{ in})(7.49\text{ in.})(3.745\text{ in}) + (10.3\text{ in.})(0.76\text{ in.})(7.87\text{ in.})$$

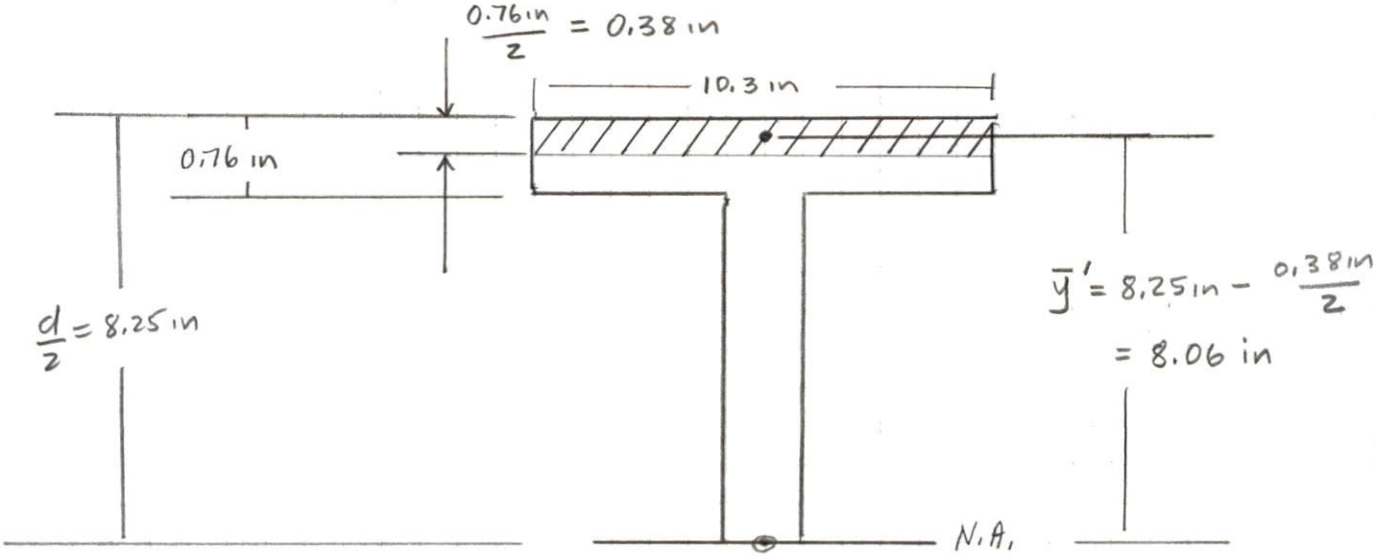
$$= 12.763\text{ in.}^3 + 61.606\text{ in.}^3$$

$$= 74.37\text{ in.}^3$$

$$\tau = \frac{VQ}{It} = \frac{(100\text{ kip})(74.37\text{ in.}^3)}{1110\text{ in.}^4\ (0.455\text{ in.})}$$

$$= \underline{\underline{14.73\text{ ksi}}}$$

(b) Shear Stress at the midpoint of Flange



$$Q = A'\bar{y}' = (10.3\,in)(0.38\,in.)(8.06\,in.) = 31.55\,in.^3$$

$$\tau = \frac{VQ}{It}$$

$$= \frac{(100\,Kip)(31.55\,in.^3)}{(1110\,in.^4)(10.3\,in.)}$$

$$= 0.28\,ksi$$

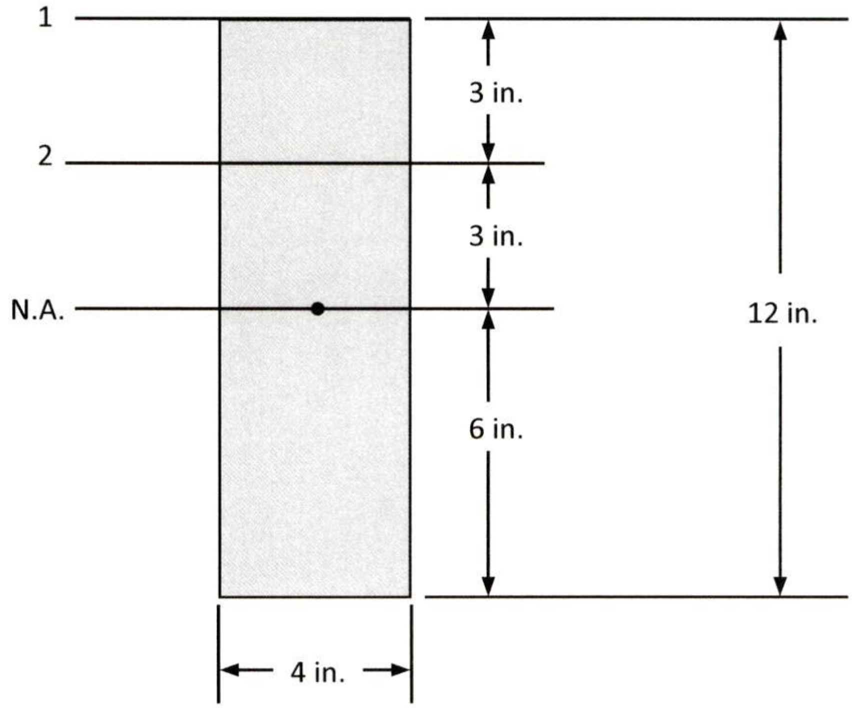
Conclusion

The shear stresses at the neutral axis are much greater than at other parts of the section.

Note: "Rounded Corners" were excluded to simplify example

Example 14-6 (Changed to U.S. System of Units)

The simple beam is subjected to a concentrated load P = 4500 lb at the midspan. The beam has the rectangular section shown. Determine the shear stresses at points along line 1, line 2, and the neutral axis. Sketch the shear stresses distribution in the section.



Solution.

Table 13-1, case 1

$$V_{MAX} = \frac{P}{2} = \frac{4500 \text{ lb}}{2} = 2250 \text{ lb}$$

Calculate the Moment of Inertia wrt the N.A.

$$I = \frac{bh^3}{12} = \frac{(4 \text{ in})(12 \text{ in})^3}{12} = 576 \text{ in}^4$$

Shear Stress
Along Line 1

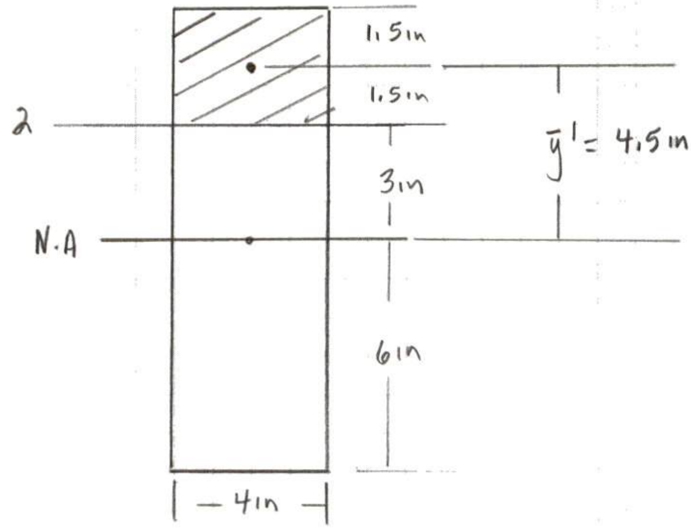
AT the top $A' = 0$

$$Q = A'\bar{y}' = 0 \Rightarrow \tau_1 = 0$$

Shear Stress Along Line 2

$$Q = A'\bar{y}'$$
$$= (4\,\text{in})(3\,\text{in})(4.5\,\text{in})$$
$$= 54\ \text{in.}^3$$

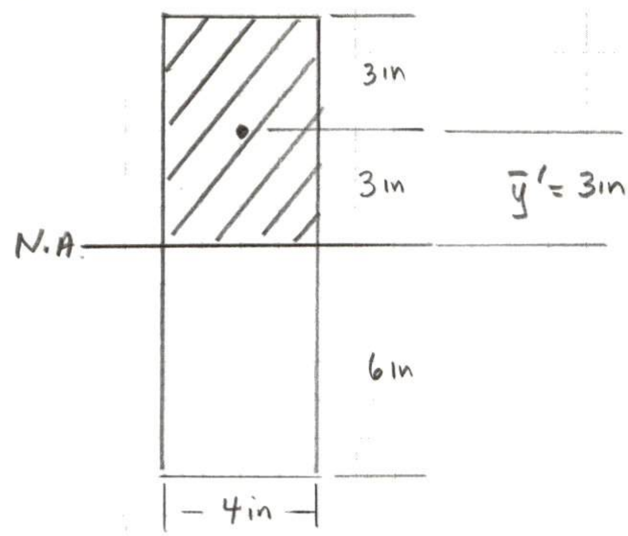
$$\tau_2 = \frac{VQ}{It} = \frac{2250\ \text{lb}\,(54\ \text{in.}^3)}{576\ \text{in}^4\,(4\,\text{in})}$$
$$= 53\ \text{psi}$$



Shear stress Along the Neutral Axis

$$Q = A'\bar{y}'$$
$$= 4\,\text{in}\,(6\,\text{in})(3\,\text{in}) = 72\ \text{in.}^3$$

$$\tau_{N.A.} = \frac{VQ}{It}$$
$$= \frac{2250\ \text{lb}\,(72\ \text{in.}^3)}{576\ \text{in.}^4\,(4\,\text{in})}$$
$$= 70\ \text{psi}$$



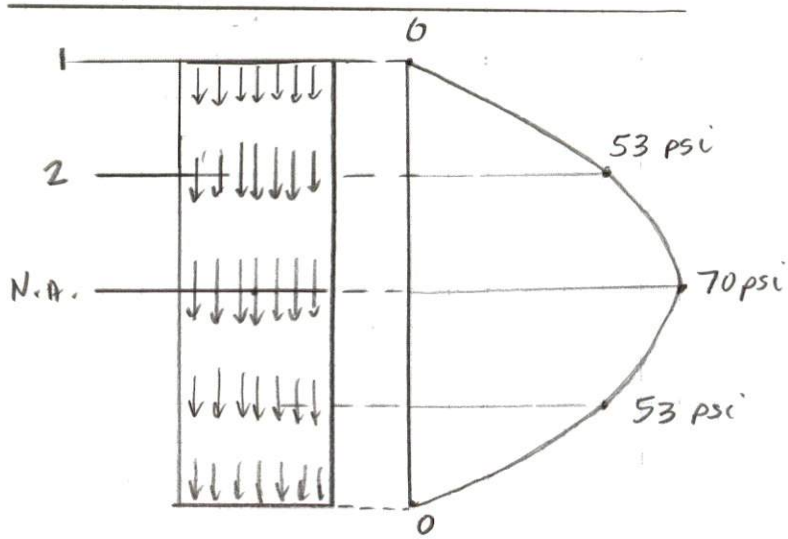
OR, EQ 14-11

AT N.A.

$$\tau_{MAX} = 1.5\,\frac{V}{A}$$
$$= 1.5\,\frac{2250\ \text{lb}}{(4\,\text{in})(12\,\text{in})}$$
$$= 70\ \text{psi}$$

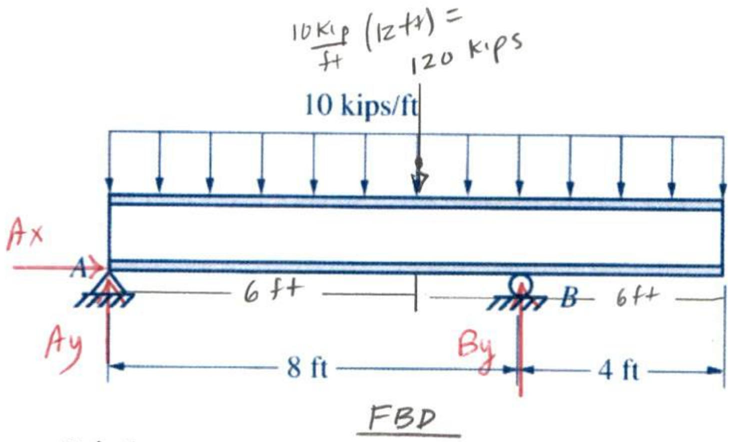
Note:
A = entire area of the rectangle

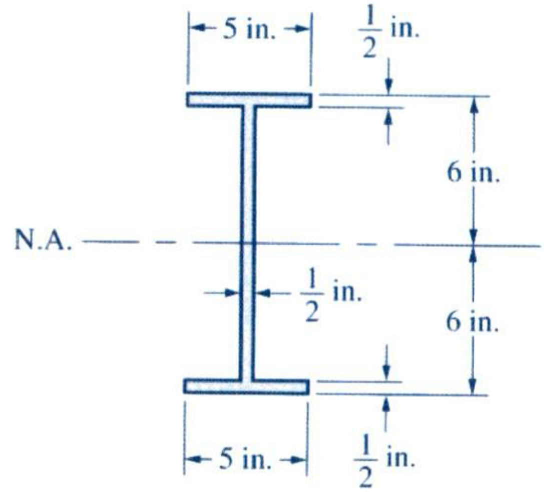
Sketch Shear Stress Distribution in the Section



Example 14-7

The overhanging beam in Fig. E14-7(1) is subjected to a uniform load as shown. The beam is built of three steel plates welded together to form an integral section, as shown in Fig. E14-7(2). Such a built-up section is called a plate girder. (a) Calculate the maximum shear stress and indicate this stress on a rectangular element at the point where it occurs. (b) At the section where the maximum shear stress occurs, calculate the shear stress at the junction of the flange and the web. (c) Plot the distribution of the shear stress for the section where the maximum shear stress occurs.





Solution.

Equilibrium Equations

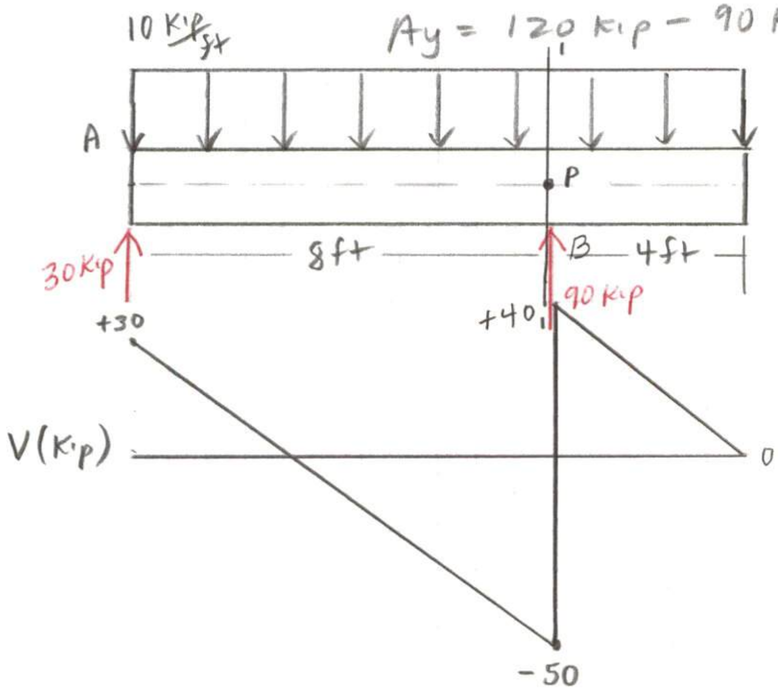
$[\Sigma F_x = 0] \quad A_x = 0$

$+\circlearrowleft [\Sigma M_A = 0] \quad -120\ \text{kips}(\text{ft}) + B_y(8\ \text{ft}) = 0$

$$B_y = \frac{720\ \text{kip}\cdot\text{ft}}{8\ \text{ft}} = 90\ \text{kip} \uparrow$$

$[\Sigma F_y = 0] \quad A_y - 120\ \text{kip} + B_y = 0$

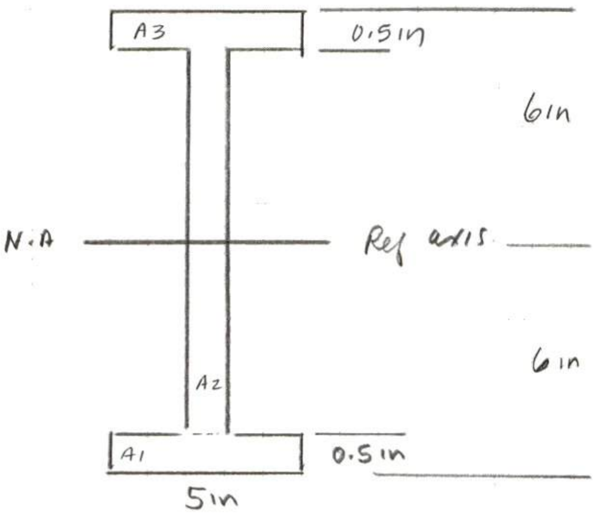
$$A_y = 120\ \text{kip} - 90\ \text{kip} = 30\ \text{kip} \uparrow$$



$$|V_{MAX}| = 50\ \text{kip}$$

Moment of Inertia of Section about the Neutral Axis

Shape	Area	$\bar{y}-y$	$A(\bar{y}-y)^2$	I
A1	2.5	5.75	82.65625	$\frac{5(0.5)^3}{12} = 0.0521$
A2	5.5	0	0	$\frac{0.5(11)^3}{12} = 55.46$
A3	2.5	5.75	82.65625	$\frac{5(0.5)^3}{12} = 0.0521$
			165.3125	



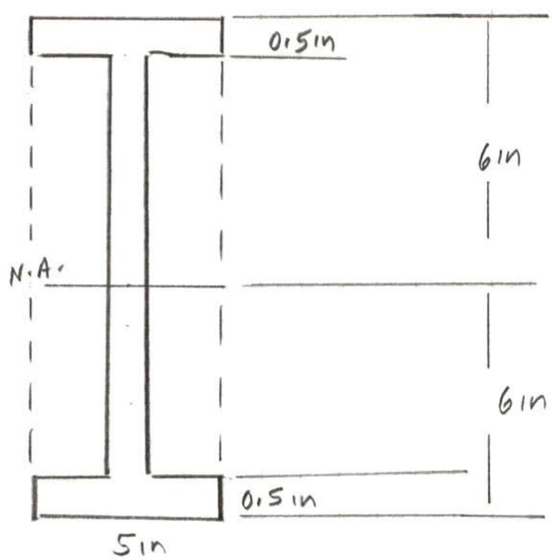
$$I = \Sigma\left[I + A(\bar{y}-y)^2\right]$$
$$= 165.3125 + 55.564$$
$$= 221 \text{ in.}^4$$

OR

Cross-sectional Area

5in × 12in Rectangle

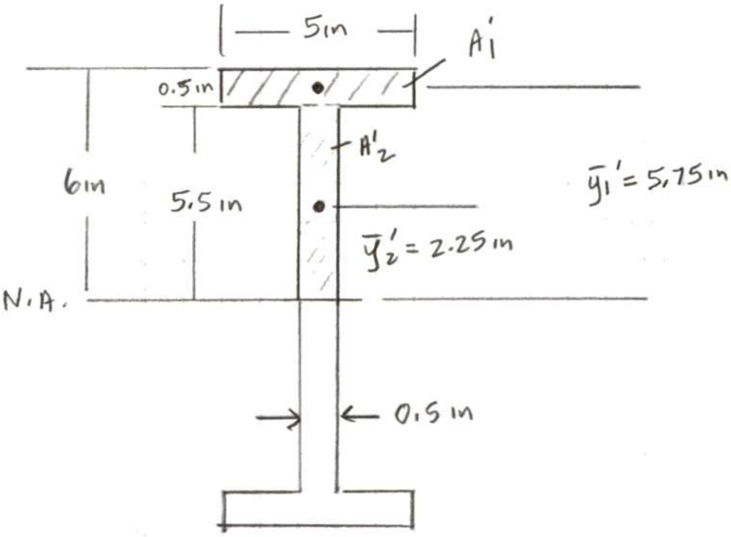
− 4.5in × 11in Rectangle



$$I = \frac{(5\text{ in})(12\text{ in})^3}{12} - \frac{4.5\text{in}(11\text{ in})^3}{12}$$
$$= 720 \text{ in}^4 - 499.125 \text{ in}^4$$
$$= 221 \text{ in.}^4$$

(a) Maximum Shear Stress occurs at P at the N.A.

$V_{MAX} = 50$ kips



$Q_1 = \bar{A}_1' \, \bar{y}_1' = 5\text{in}(0.5\text{in})(5.75\text{ in})$

$= 14.38 \text{ in.}^3$

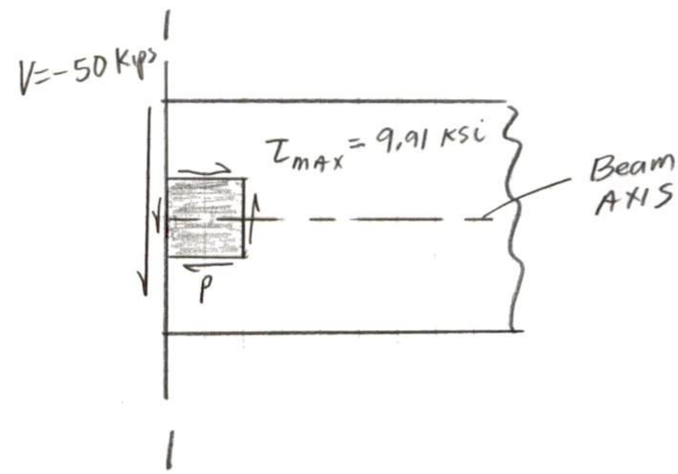
$Q_2 = \bar{A}_2' \, \bar{y}_2' = 0.5\text{in}(5.5\text{in})(2.25\text{ in})$

$= 7.56 \text{ in.}^3$

$Q = Q_1 + Q_2 = 21.94 \text{ in.}^3$

$$\tau_{MAX} = \frac{VQ}{It} = \frac{(50\text{ kips})(21.94\text{ in.}^3)}{221\text{ in.}^4\,(0.5\text{ in})} = 9.91\text{ ksi}$$

(b) Shear stress at the junction of the flange and the web



At the junction of the flange and the web,

$t = 5$ in

or $t = 0.5$ in

Area of the flange A_1'

$Q = Q_1 = 14.38 \text{ in}^3$

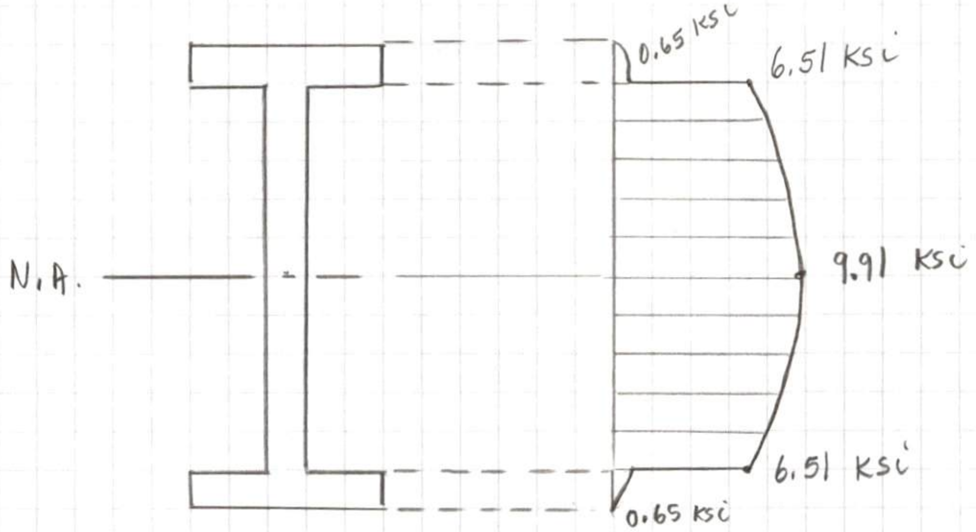
Shear stress in the flange where $t = 5\text{in}$

$$\tau_{flange} = \frac{VQ}{It} = \frac{50\text{ kips}\,(14.38\text{ in.}^3)}{221\text{ in}^4\,(5\text{in})} = 0.651\text{ ksi}$$

When $t = 0.5$ in

$$\tau_{web} = \frac{VQ}{It} = \frac{50\text{ kips}\,(14.38\text{ in.}^3)}{221\text{ in}^4\,(0.5\text{ in})} = 6.51\text{ ksi}$$

(c) Shear Stress Distribution



Average Web Shear

From example 14-7 we can see that most of the shear force is resisted by the web in a wide-flange setion.

Design codes, such as American Institute of Steel Construction (AISC) specifications allow the use of an average web shear to calculate the maximum shear stress in fabricated or hot-rolled wide-flange or I-beam sections.

$$\tau_{avg} = \frac{V_{max}}{dt_w}$$

where,

τ_{avg} = the average shear stress in the web

V_{max} = the maximum shear force along the beam

d = the FULL depth of the beam

t_w = the web thickness of the beam section